Chapter 20

Principal Protection Techniques

1. Introduction

Investment products, where the principal is protected, have always been popular in financial markets. However, until recently the so-called guaranteed products sector has relied mainly on *static* principal protection which consists of a static portfolio of a default-free bond plus a basket of options. The advances in financial engineering techniques recently changed this. As structurers understood dynamic replication better, they realized that dynamic replication could synthetically create options on risks where no traded options exist. This led to the creation of dynamic rebalancing techniques, the best known being the constant proportion portfolio adjustment (CPPI). With the development of credit markets, CPPI was applied to credit indices and the implied tranches as well. As expertise on the dynamic replication techniques grew, market activity led to new innovations such as the dynamic proportion portfolio protection (DPPI).

A major reason for the popularity of the guaranteed product sector is regulatory behavior. Several countries do not let investment banks issue structured products involving "exotic" risks unless the product provides some principal protection guarantee. CPPI is a protected note and is not subject to these restrictions. In addition, many funds are not allowed, by law, to invest in securities that are speculative grade. Other dynamic proportion techniques are not principal protected, but both the principal and the coupon can be rated AAA by the main rating agencies, although they offer unusually high coupons. These make them attractive to conservative funds as well.[1]

In this chapter we discuss the classical *static* principal protection methodology followed by an introduction to the CPPI as an extension of this classical methodology. We show the dynamics of the portfolio value that applies this technique under some simple conditions and provide the

[1] We call these guaranteed products. It is important to realize that this guarantee was initially toward market risk; the classical guaranteed products could still carry a credit risk. The (institutional) investor could conceivably lose part of the investment if default occurs.

results of some simulations as well. We then deal with the application of the methodology to standard credit index tranches and discuss the complications that may arise when this is implemented. Finally, we introduce some modeling aspects and discuss the so-called *gap risk*, and deal briefly with the DPPI.

2. The Classical Case

At the simplest level the guaranteed product consists of a zero coupon bond and one or more options.[2] Suppose S_t denotes the value of an underlying security. This security can essentially be anything from stocks to credit index tranches, or the value of some hedge fund investment. We can then write the following contractual equation:

$$\boxed{\begin{matrix}\text{Guaranteed product} \\ \text{with } S_t \text{ exposure}\end{matrix}} = \boxed{\begin{matrix}\text{A zero coupon} \\ \text{bond}\end{matrix}} + \boxed{\begin{matrix}\text{Long option} \\ \text{with } S_t \text{ exposure}\end{matrix}} \tag{1}$$

Suppose an investor invests the amount $N = 100$ directly to a basket of options over a T-year maturity. Then, options being risk investments and investors having limited risk management capabilities, part of the principal may be lost if these options expire out-of-the-money. On the other hand, if the yield on a T-maturity zero coupon bond is r% and if the same investor invests, at time t_0, a carefully chosen amount PV_{t_0} in this bond, the security will be worth 100 in five years:

$$PV_{t_0}\left(1 + r_{t_0}\right)^T = N \tag{2}$$

Thus the investor can allocate PV_{t_0} to buy a bond, and will still have $N - PV_{t_0}$ to invest in (a basket of) options. Depending on the level of volatility, the level of r and the expiration dates under consideration, this residual will provide an exposure—the growth of S_t. In fact, let g_{t_i} be the percentage rate of change in S_t during the interval $[t_i, t_{i-1}]$,

$$g_{t_i} = \frac{S_{t_i} - S_{t_{i-1}}}{S_{t_{i-1}}} \tag{3}$$

Then the investor's exposure will be $\lambda g_{t_i} S_{t_0}$, where λ is the familiar *participation rate*. In the case of structured products, it is the bank that makes all these calculations, selects a structure with a high participation rate, and sells the principal protected security as a package to the investor.

According to this, in the simplest case the bank will buy a PV_{t_0} amount of the zero coupon bond for every invested 100, and then options will be purchased with the rest of the principal.

EXAMPLE:

An investor has the principal $N = 100$. The observed yield on a five-year zero coupon Treasury is 4.75%. If the investor invests 79.21 in this bond, the security will be worth 100 in five years:

$$79.21\,(1 + .0475)^5 = 100 \tag{4}$$

Thus the investor can allocate 79.21 to buy a bond, and will still have 20.7079 to invest in options.[3]

[2] Thus regulation of this sector can be visualized as demanding principal protection before options-related products are sold to retail investors.

[3] In real life there are also structurers' fees that need to come out of this amount. These fees are collected up front.

The general idea is simple: The problems arise in implementation and in developing more refined ways of doing this. In practice several problems can occur.

First, zero coupon interest rates for a maturity of T may be too low. Then the zero coupon bond may be too expensive and not enough "excess" may be left over to invest in options. For example, during the years 2002–2004, five-year USD Treasury rates were around 2%. This leaves only:

$$100 - \frac{100}{(1+.02)^5} = 9.42 \tag{5}$$

to invest in the option basket. Once we factor out the fees paid for such products, which could be several percentage points, the amount that can be invested in the option goes down even more.[4] Depending on the level of volatility, such an investment may not be able to secure any meaningful participation rate.

Second, options on the underlying where exposure is desired may not exist. For example, considering hedge funds, there are few options traded on these risks. Yet, an investor may want exposure to hedge fund activity, or, say, to credit tranches without risking (part of) his or her principal.

Third, irrespective of the level of interest rates, the options may be too expensive, depending on the level of volatility. This may, again, not secure a meaningful participation rate.

These problems have led to several modifications of the traditional principal protection methods. Dynamically adjusted principal protection methods (versions of CPPI) and the dynamically adjusted methods that yield triple-A products have been developed as a result. We study the CPPI techniques first and discuss their application to standard credit tranches.

3. The CPPI

The main advantage of the CPPI as a principal protection technique is that it gives a higher participation in the underlying asset than one can get from traditional capital protection. It also can be applied when interest rates are "too" low, or when options do not trade for some underlying risk. Before we discuss the CPPI algorithm and the associated risks we consider some market examples.

EXAMPLE:

The CPPI investment is an alternative to standard tranche products, which offer limited upside (fixed premium) in exchange for unlimited downside (potential total loss of principal). CPPI offers limited downside (because of principle protection) and unlimited upside, but exposes investors to the market risk of the underlying default swaps contracts that comprise the coupon.

With yields hovering near record lows until recently, credit investors are increasingly moving to structures that contain some element of market exposure. That has posed a problem for ratings agencies, which are default oriented, and prompted a move to a more valuation-based approach for some products. Leveraged super senior tranches, also subject to market volatility, were the market-risk product of choice last year, but in recent months have ceded popularity to CPPI.

[4] An estimate of the average fees for structured products is provided by IFR, March 12, 2007. According to Thompson the average fees range between 30 and 100 bps in Europe and between 60 and 150 bps in the United States.

Banks profit from ratings on CPPI coupons because the regulatory capital treatment of rated products under Basel II is much kinder than on unrated holdings.

The CPPI is a structure which has constant *leverage*. Dynamic PPI is the name for strategies where the leverage ratio changes during the investment period. CPPI works by dynamically moving the investment between a safe asset and a risky asset, depending on the performance of the risky asset and depending on how much cash one has in hand. The main criteria in doing this is to protect the principal, while at the same time getting the highest participation rate.

The idea of CPPI can be related to the classical principal protection methods and is summarized as follows. In the classical principal protection the investor buys a zero coupon bond and invests the remaining funds to options. CPPI relaxes this with a clever modification. If the idea is to be long a bond with value 100 at T, then one can invest *any* carefully selected sum to the risky asset as long as one makes sure that the total value of the portfolio remains above the value of the zero coupon bond during the investment period. Then, if the portfolio value is above the value of the zero coupon bond, at any desired time the risky investment can be liquidated and the bond bought. This will still guarantee the protection of the principal, N. This way, the structurer is not limited to investing just the leftover funds. Instead, the procedure makes possible an investment of funds of any size, as long as risk management and risk preference constraints are met.

Let the initial investment of N be the principal. The principal is also the initial net asset value of the positions—call it V_{t_0}. Next, calculate the present value of N to be received in T years. Call this the *floor*, F_{t_0}.[5]

$$F_{t_0} = \frac{N}{(1 + r_{t_0})^T} \tag{6}$$

Let the increment Cu_t be called the *cushion*:

$$Cu_{t_0} = V_{t_0} - F_{t_0} \tag{7}$$

Then, select a *leverage ratio* λ in general satisfying $1 < \lambda$. This parameter has no time subscript and is constant during the life of the structured note. Using the λ calculate the amount to be invested in the *risky asset* R_{t_0} as:

$$R_{t_0} = \lambda \left[V_{t_0} - F_{t_0} \right] \tag{8}$$

This gives the *initial* exposure to the risky asset S_t. Invest R_{t_0} in the risky asset and deposit the remaining $V_{t_0} - R_{t_0}$ into a risk-free deposit account.[6]

$$D_{t_0} = V_{t_0} - R_{t_0} \tag{9}$$

Note that the cash deposit D_{t_0} is less than the time-t_0 value of a risk-free zero coupon bond that matures at time T, denoted by $B(t_0, T)N$, where $B(t_0, T)$ is the time t_0 price of a default-free discount bond with par value \$1. Hence, in case of a sudden and sizable downward *jump* in S_t, the note will not have enough cash in hand to switch to a zero coupon bond. This is especially true if the jump in S_t leads to flight-to quality and increases the $B(t_i, T)$ at some future date t_i. This is called the *gap risk* by the structurers and is studied later in the chapter.

[5] If there are fees, then they should be deducted at this point from the F_{t_0}. In reality, there are always such fees, but in this discussion we assume that they are zero to simplify the exposition.

[6] If $R_{t_0} < V_{t_0}$, then the risky asset investment does not require any additional borrowing. If, on the other hand, $R_{t_0} > V_{t_0}$, then invest R_{t_0} in the risky asset and borrow the remaining $R_{t_0} - V_{t_0}$.

Apply this algorithm at every rebalancing date t_i,

$$t_i - t_{i-1} = \delta_i \tag{10}$$

as long as $F_{t_i} < V_{t_i}$. This algorithm would increase the investment in the risky asset if things go well (i.e., if V_{t_i} increases), and decrease the investment in case markets decline (i.e., if V_{t_i} decreases).

Finally, if at some time τ^* V_{τ^*} falls and becomes equal to F_{τ^*}, liquidate the risky investment position and switch all investment to cash. Since the floor F_{t_i} is the present value at t_i, of 100 to be received at T, this will guarantee that principal N can be returned to the investors at T.

4. Modeling the CPPI Dynamics

We now obtain the equations that give the dynamics of V_t under the typical CPPI scheme. The equations are obtained from a relatively simple setting to highlight the important aspects of the methodology. The two points on which we focus are the following: First, we will see that from a single portfolio point of view, CPPI algorithms may be much more stable than they appear from the outside if there are no jumps. However, in reality there are jumps which lead to the gap risk. Second, we show that the CPPI methodology may have a more fragile structure with respect to yield curve movements than anticipated. This may be especially the case if sharp downward jumps in S_t are *correlated* with a sudden steepening of the curve—exactly what happens during periods of excessive market stress.[7]

Let us place ourselves in a Black-Scholes type environment, with constant interest rates r and constant volatility σ. Further, the S_t follows the Wiener process-driven SDE,

$$dS_t = \mu S_t dt + \sigma S_t dW_t \tag{11}$$

We know from the previous discussion that the investment in risky assets is

$$R_t = \lambda C u_t \tag{12}$$

and that the changes in the cushion are given by

$$dCu_t = d(V_t - F_t) \tag{13}$$

This means

$$dCu_t = (V_t - R_t)\frac{dB_t}{B_t} + R_t\frac{dS_t}{S_t} - dF_t \tag{14}$$

where B_t is the value of the zero coupon bond at time t.[8] In this expression we can replace V_t, R_t with their respective values to obtain

$$dCu_t = (Cu_t + F_t - \lambda Cu_t)\frac{dB_t}{B_t} + \lambda Cu_t\frac{dS_t}{S_t} - dF_t \tag{15}$$

But the value of the floor increases according to

$$dF_t = F_t\frac{dB_t}{B_t} \tag{16}$$

[7] As CPPI type protection techniques became more popular, academic interest also increased. The recent work by Cont and Tankov (2007) and Cont and El Kaouri (2007) provides an excellent view of the academic approach to this issue and is also quite accessible and practical. This section is based on this research.

[8] In this case this value is given by the simple formula due to constant r, $B_t = B_{t_0}e^{r(t-t_0)}$.

This means

$$dCu_t = (1-\lambda)Cu_t\frac{dB_t}{B_t} + \lambda Cu_t\frac{dS_t}{S_t} \tag{17}$$

Substituting further,

$$dCu_t = (\lambda(\mu - r) + r)Cu_t dt + \lambda\sigma c dW_t \tag{18}$$

This is a *geometric* stochastic differential equation whose solution is given by

$$Cu_t = Cu_{t_0} e^{\left(\lambda(\mu-r)+r-\frac{\lambda^2\sigma^2}{2}\right)(t-t_0)+\lambda\sigma W_t} \tag{19}$$

We can combine this with F_{t_0} get the behavior of the portfolio net asset value for all $t \in [t_0, T]$

$$V_t = F_{t_0}e^{r(t-t_0)} + Cu_t \tag{20}$$

Using standard results from stochastic calculus we can calculate the expected portfolio value as of time t

$$E^P_{t_0}[V_t] = F_{t_0} + \left(100 - F_{t_0}e^{r(T-t_0)}\right)e^{r\left(t-t_0\right)+\lambda(\mu-r)\left(t-t_0\right)} \tag{21}$$

Note that the probability measure we use in this expectation is the real-world probability and not the risk-adjusted measure. This is the case since the drift of the S_t process was taken to be the μ and not the risk-free rate r.

4.1. Interpretation

There are two equations in the above derivation that are very suggestive. The first relates to the dynamics of the cushion over time,

$$dCu_t = (\lambda(\mu - r) + r)\, Cu_t dt + \lambda Cu_t\frac{dS_t}{S_t} \tag{22}$$

Note that with such a dynamic the cushion itself can never go below zero over time. In fact, suppose Cu_t becomes very small at time t. The first term on the right-hand side of the equation will be positive. Also, being a Wiener-driven system, the S_t cannot exhibit jumps and over infinitesimal periods must change infinitesimally. The second term on the right-hand side then shows that the changes in S_t affect the cushion with a coefficient of λCu_t, which goes to zero as Cu_t approaches zero. Under these conditions, as the cushion Cu_t goes toward zero, the dCu_t will go to zero as well.

This leads to an interesting conclusion. The CPPI method will always "work" in the sense that as the market goes against the investor, the cushion will never become negative. In the worst case, the cushion will be zero which means that the risky investment is liquidated. This leaves the investor with the zero coupon bond which matures at a value of 100. Hence, the principal is "always" protected.

Before we continue with the implications of this result consider the second interesting equation. The expected value of the portfolio was calculated as

$$E^P_{t_0}[V_t] = F_{t_0} + \left(100 - F_{t_0}e^{r\left(T-t_0\right)}\right) e^{r\left(t-t_0\right)+\lambda(\mu-r)\left(t-t_0\right)} \tag{23}$$

This is also very suggestive because, as long as $r < \mu$, which means that the risky asset expected return is higher than the risk-free rate, the expected value of the portfolio can be increased indefinitely by picking higher and higher leverage factors, λ.

Thus we have reached an unrealistic result. The CPPI strategy will always work, in the sense that the investor's initial investment is always protected, while at the same time the higher the leverage the higher the expected gains. This implies picking the highest possible leverage factor that is available to the structurer. Yet, it is clear that in the real world this is not the prudent approach. This, in turn, suggests that the model we discussed above may be missing some critical features of real-world investment.

There are at least two possibilities, the first being the limits on borrowing. There are credit limits and the leverage cannot be increased indefinitely in the real world. The second possibility is more interesting.

In the real world, the S_t process may not follow a geometric process and may contain jump factors. If this is the case, as $Cu_t \to 0$, a downward jump in S_t can make Cu_t negative. The portfolio value will fall *below* the floor and the investor's initial investment is lost,

$$V_t < F_t \tag{24}$$

This is the *gap risk*. Note that, if there are such jumps in the S_t, then the presence of the leverage factor $1 < \lambda$ will magnify them. The higher the λ the higher will be the effect of a downward jump.

4.2. How to Pick λ

The discussion involving the gap risk suggests a methodology for selecting a numerical value for the critical leverage parameter λ. The structurer would first determine an acceptable threshold for the gap probability using the investor's risk preferences. Then, using the observed volatility and jump parameters, the structurer would work backward and determine the λ that makes the gap probability equal to this desired amount. This could be done with Monte Carlo, or with semiparametric methods as in Cont and Tankov (2007).

Clearly this determination of λ will be model dependent. A structurer would have a number of other ways to deal with this gap risk, some of which are discussed below.

5. An Application: CPPI and Equity Tranches

We will use structured credit as an application of the CPPI technique. The evolution of the credit sector has been very impressive. Two paradigms are observed in the structured credit sector: The first tracks some credit derivatives index, and the second is managed credit derivatives funds. CPPI techniques can be useful in both of these trends as shown in the example from the markets.

EXAMPLE:

Retail credit CPPI first ABN AMRO and AXA last week announced that they had closed the first principal protected credit derivatives fund targeted at retail investors. AXA persuaded its home regulator in France to permit the leveraged fund, and similar deals are expected to be launched in other regimes.

The fund is structured like an actively managed synthetic collateralized debt obligation. It will have a minimum of 100 credit default swap references and is starting with just under 120 names.

It uses constant proportion portfolio insurance from ABN AMRO to provide the capital protection that was necessary to obtain regulatory approval for its sale to retail investors in France. The insurance is provided on a binary basis if the basket of default swaps *managed by AXA performs so poorly that a zero-coupon bond would have to be bought to guarantee investor capital, then there would be an effective wind-up.*

In this section we discuss the application of the CPPI technique to standard credit index tranches. It turns out that combining CPPI and iTraxx Tranches is quite simple in terms of financial engineering, although there are some practical issues that need to be resolved in practice. Here is the algorithm.

1. Receive cash of $N = 100$ from the investor.
2. Calculate the difference between par and the cost of a zero coupon bond F_t, and as before let this term be Cu_t, the cushion.
3. Multiply the Cu_t by the leverage factor λ; this is the amount to be invested in the risky asset, which in this case is the iTraxx 0–3% equity tranche. Keep the remaining cash in a deposit account or as collateral.[9]
4. As the price of the iTraxx equity tranche goes up and down, adjust the allocation between risky asset and reserve cash to keep the leverage of the trade constant at λ.

Thus far this is a straightforward application of the previously discussed CPPI algorithm. The only major complication is in the definition of the risky asset. Standard equity tranches are quoted as up front percentages and this will lead to a modification of the algorithm. In fact, suppose the amount to be invested in the risky asset is R_t, and suppose also that the iTraxx equity tranche quote is given by the q_t where the latter is a pure number denoting the upfront payment as a percentage.[10] Then the relationship between the amount allocated in the risky asset and the notional amount invested in the equity tranche N^{Eq} will be as follows,

$$N_{t_i}^{Eq} = \frac{R_{t_i}}{(1 - q_t)} \tag{25}$$

Note that once we take the upfront fee into account the exposure will be R_{t_i},

$$R_{t_i} = N^{Eq} - \frac{R_{t_i}}{(1 - q_t)} q_t \tag{26}$$

It may be helpful to discuss a numerical example at this point.

5.1. A Numerical Example

First we note that the equity tranche of the iTraxx Index is quoted as an upfront percentage of the notional plus 500 basis point *running-fee* paid quarterly. Assume that the upfront fee is $q_t = 20\%$ of the notional N and that the annual Libor rate is $L_t = 5\%$. For simplicity, suppose the swap curve is flat at 5.095% as well. The CPPI is applied with daily adjustment periods denoted by $t_i,\ i = 0, 1, \ldots, n$. Assume no bid-ask spreads.

We apply the steps above in a straightforward fashion to a five-year CPPI note where the underlying is the equity tranche.

5.1.1. The Initial Position

Initially the CPPI will have the following structure.

[9] Remember that the iTraxx indices are unfunded.

[10] For example if a market maker quotes $q_t = 12\%/12.5\%$, then a protection seller will receive \$12 up front for each 100 dollars of insurance sold. For the protection seller, this money is to keep.

1. Receive $N = 100$.
2. The floor is

$$F_t = \frac{100}{(1+.05095)^5} \\ = 78 \tag{27}$$

3. The cushion is

$$Cu_t = 100 - 78 \\ = 22 \tag{28}$$

4. Assuming a leverage of

$$\lambda = 2 \tag{29}$$

   the amount to be invested in the risky asset is

$$22 \times 2 = 44 \tag{30}$$

   This is the investment to be allocated to the equity tranche.
5. The iTraxx equity tranche pays an upfront cash amount of $20\% \times N$. Therefore, if the risky asset exposure is $R_t = 44$, then the notional amount invested in the equity tranche N^{Eq} should be[11]

$$N^{Eq} = \frac{44}{(1-.20)} = 55 \tag{31}$$

   Thus, sell equity tranche protection with notional 55, and then the balance of USD100 in a default-free deposit account, receiving Libor. The balance of 56 is kept in this account and receives Libor. Note that the total amount of cash to be kept as collateral for the equity protection position is

$$q_t N^{Eq} + (100 - \lambda Cu_t) = 11 + 44 = 55 \tag{32}$$

   Confirming that the 44 is in the risky asset.
6. As the equity tranche quote changes over the rebalancing periods $t_1, t_2, \ldots$ adjust the position dynamically, reducing the exposure to risky asset as q_t increases, and increasing the exposure as q_t decreases.

Note that during this process the equity tranche position is actually taken as an unfunded investment. Still, the cash allocated to the risky asset, plus the upfront cash, is held as collateral for the position.

5.1.2. Dynamic Adjustments

Let, for example, $q_{t_1} = 15\%$. This corresponds to an increase in the value of the risky asset investment. After all, one can buy protection at 15% and close the position with a profit. Yet, with the structured note the position is continued after an adjustment. We cover these steps below.

[11] In other words, if we invest 55 in the equity tranche notional then our net exposure to the risky asset will be 44, since we did get 11 as upfront.

1. The value of the risky asset is

$$
\begin{aligned}
N\,(1 - q_{t_1}) &= 55.85 \\
&= 46.75
\end{aligned} \tag{33}
$$

The value of the risky asset has gone up by

$$
R_{t_1} - R_{t_0} = 2.75 \tag{34}
$$

This is the case since we can close the position by buying equity protection at 15% up front. Then we recover the 44 deposited as collateral for the equity tranche investment, and in addition we receive the realized gain

$$
55.05 = 2.75 \tag{35}
$$

by not losing this position, it is as if we are investing 46.75 in the risky asset.

2. Calculate the V_{t_1} using

$$
V_{t_1} = F_{t_0}(1 + L_{t_0}) + N_{t_0}(q_{t_1} - q_{t_0}) + N_{t_0}(1 + L_{t_0}) + N(.05) \tag{36}
$$

In this case this amounts to

$$
105.095 + 2.75 \;+\; (11)(0.05) \;+\; 55(.05) \tag{37}
$$

The first term is the interest on the 100 kept as cash or collateral. The second term is the capital gains from the q_t move, the third is the Libor earned from the upfront deposit. Finally the fourth term is the 500 bp running fee on the 55.

The opposite adjustment will be implemented if the q_{t_1} decreases. We leave the details of this case to the reader. Instead, we will consider the case of a default.

One can claim that the CPPI strategy is dynamically replicating a long option position. The long option position is also long volatility and hence the higher the volatility the better things are for the option holder. In other words, the option itself can be replicated by a dynamic adjustment technique that leads to an increase in the instantaneous volatility.

6. A Variant: The DPPI

Dynamic portfolio insurance (DPPI) methodology is a variation of the CPPI. Here is a brief example from the markets.

EXAMPLE:

LONDON, June 14 (Reuters)—PIMCO, one of the world's biggest bond funds, has joined forces with Goldman Sachs to launch a range of derivative products. The investments include principal protected and leveraged structures aimed at institutional investors, high net worth individuals and private banks.

The main product, launched under PIMCO is a principle protected investment based on Goldman's Variable Proportion Portfolio Protection, similar to the better-known CPPI technology.

The leverage ratio λ_{t_i} which was constant during the CPPI adjustments can be made variable and becomes one of the unknowns to be determined. The structurer needs to provide an algorithm to do this. The idea is that the leverage ratio can be made to depend on some variables that one thinks are relevant to the problem under consideration in some optimal fashion. In particular, the exposure to the risky asset may depend on

1. The past behavior of the returns,
2. The volatility of the returns,
3. The liquidity observed in the market for the underlying asset, since the methodology is heavily dependent on the correct rebalancing,
4. And upon the so-called *gap-risk*.

Finally, another relevant variable may the dependence of λ_{t_i} on the swap curve parameters. The scenarios discussed above illustrated the importance of this. Note that this may be even more relevant for the credit market CPPI notes.

In DPPI the allocation between the risky asset and cash is dynamically managed with a variable leverage ratio that will depend on one or more of these factors. Supposedly, the CPPI exposure to the risky asset increases when things "go well," and decreases when things "go badly." At the outset, a variable leverage ratio seems to be better able to handle changes in the yield curve environment than the classical CPPI procedures. For example, the leverage ratio λ_{t_i} may go down during *high* volatility periods, and may go *up* during low volatility periods. The response to changes in the market-liquidity could be similar.

7. Real-World Complications

The idea behind CPPI techniques is simple. Actually, even the modeling is fairly straightforward. Yet, in practice, several diffculties arise. We will look at only some of them.

7.1. The Gap Risk

If a structurer does not want exposure to gap risk then it could be sold to other investors through other structured products. For example, with structured products such as autocallables, a high coupon is paid to the investor, but the structure is called automatically if the underlying price hits a preselected level. Note that with autocallables the investor receives a high coupon but also assumes the risk of large downward movements in a basket. The extra coupon received by the investor can be visualized as the cost of insuring the *gap risk*.[12]

Another possibility frequently used in practice is to manage the gap risk using deep out-of-the-money puts. This is possible if the underlying is liquid. However, in the case of CPPI strategies, the underlying is often illiquid and this makes *delta*-hedging of the option positions difficult. Still, one can claim that during stress periods, correlations go toward one and liquid indices can become correlated with the illiquid underlying. Hence, carefully chosen deep out-of-the-money options on liquid indices can also hedge the gap risk.

7.2. The Issue of Liquidity

The issue of liquidity of the underlying is important to CPPI-type strategies for several reasons. First, is the need to close the risky asset position when the cushion goes toward zero. If the

[12] Alternatively, the gap risk can be insured by a reinsurance company.

underlying market is not liquid this may be very difficult to do, especially when markets are falling at a steep rate.[13]

Second, if the underlying is illiquid, then options on the underlying may not trade and hedging the gap risk through out-of-the-money options may be impossible.

The third issue is more technical. As mentioned in the previous section, gap risk can be modeled using the jump process augmented stochastic differential equations for the S_t. In this setting jump risk is the probability that Cu_t is negative. This determines the numerical value selected for the λ parameter. However, note that if the underlying is not liquid, then options on this underlying will not be liquid either. Yet, liquid option prices are needed to calibrate the parameters of the jump process. With illiquid option markets this may be impossible. Essentially the selection of λ would depend on arbitrarily made assumptions and/or historical data.

8. Conclusions

There may be several other real-world complications. For example, consider the application of the CPPI to the credit sector. One very important question is what happens on a *roll*? Clearly the structurer would like to stay with on-the-run series, and during the roll there will be mark-to-market adjustments which may be infinitesimal and similar to jumps.

A second question is how to pick the leverage factor in some optimal fashion. It is clear that this will involve some Monte Carlo approach but the more difficult issue is how to *optimize* this.

Suggested Reading

There are relatively few sources on this topic. We recommend strongly the papers by ***Cont and Tankov*** *(2007) and* ***Cont and El Kaouri*** *(2007). The original paper by* ***Black*** *(1989) can also be consulted.*

[13] Note that the CPPI strategy will enhance the market direction. The structurer will sell (buy) when markets are falling (rising). Hence at the time when the risky asset position needs to be liquidated, other CPPI structurers may also be "selling."

Exercises

1. We consider a reference portfolio of three investment grade names with the following one-year CDS rates:

$$c(1) = 116$$
$$c(2) = 193$$
$$c(3) = 140$$

   The recovery rate is the same for all names at $R = 40$.

   The notional amount invested in every CDO tranche is \$1.50. Consider the questions:

   (a) What are the corresponding default probabilities?
   (b) How would you use this information in predicting actual defaults?
   (c) Suppose the defaults are uncorrelated. What is the distribution of the number of defaults during one year?
   (d) How much would a 0–66% tranche lose under these conditions?
   (e) Suppose there are two tranches: 0–50% and 50–100%. How much would each tranche pay over a year if you sell protection?
   (f) Suppose all CDS rates are now equal and that we have $c(1) = c(2) = c(3) = 100$. Also, all defaults are correlated with a correlation of one. What is the loss distribution? What is the spread of the 0–50% tranche?

2. The iTraxx crossover index followed the path given below during three successive time periods:

$$\{330, 360, 320\}$$

   Assume that there are 30 reference names in this portfolio.

   (a) You decide to select a leverage ratio of 2 and structure a *five-year* CPPI note on iTraxx crossover index. Libor rates are 5%. Describe your general strategy and, more important, show your initial portfolio composition.
   (b) Given the path above, calculate your portfolio adjustments for the three periods.
   (c) In period four, iTraxx becomes 370 and one company defaults. Show your portfolio adjustments. (Assume a recovery of 40%. Reminder: Do not forget that there are 30 names in the portfolio.)

3. We consider a reference portfolio of four investment grade names with the following one-year CDS rates:

$$c(1) = 14$$
$$c(2) = 7$$
$$c(3) = 895$$
$$c(4) = 33$$

   The recovery rate is the same for all names at $R = 30\%$.

The notional amount invested in every CDO tranche is $1.00. Consider the questions:

(a) What are the corresponding annual default probabilities?
(b) Suppose the defaults are *uncorrelated*, what is the distribution of the number of defaults during one year?
(c) Suppose there are three tranches:

- 0–50%
- 50–75%
- 75–100%

How much would each tranche pay over a year?
(d) Suppose the default correlation becomes 1, and all CDS rates are equal at 60 bp, answer questions (a)–(c) again.
(e) How do you hedge the risk that the probability of default will go up in the equity tranche?

4. Consider the following news from Reuters:

1407 GMT [Dow Jones] LONDON—According to a large investment bank investors can boost yields using the following strategies:

(1) In the strategy, sell 5-yr CDS on basket of Greece (9 bp), Italy (8.5 bp), Japan (4 bp), Poland (12 bp) and Hungary (16 bp), for 34 bp spread. Buy 5-yr protection on iTraxx Europe at 38 bp to hedge.

Trade gives up 4 bp but will benefit if public debt outperforms credit.

(2) To achieve neutral or positive carry, adjust notional amounts—for example in the first trade, up OECD basket's notional by 20% for spread neutral position.

(3) Emerging market basket was 65% correlated with iTraxx in 2005, hence use the latter as hedge.

(a) Explain the rationale in item (1). In particular, explain why the iTraxx Xover is used as a hedge.
(b) Explain how you would obtain positive carry in (2).
(c) What is the use of the information given in statement (3)?

5. Consider the following quote:

Until last year, this correlation pricing of single-tranche CDOs and first-to-default baskets was dependent on each bank or hedge fund's assessment of correlation. However, in 2003 the banks behind iBoxx and Trac-x started trading tranched versions of the indexes. This standardization in tranches has created a market where bank desks and hedge funds are assessing value and placing prices on the same products rather than on portfolios bespoke single-tranche CDOs and first-to-default baskets. Rather than the price of correlation being based on a model, it is now being set by the market.

(a) What is the iTraxx index?
(b) What is a *standard* tranche?

(c) Explain the differences between trading standardized tranches and the tranches of CDOs issued in the market place.

6. We consider a reference portfolio of three investment grade names with the following one-year CDS rates:

$$c(1) = 56$$
$$c(2) = 80$$
$$c(3) = 137$$

The recovery rate is the same for all names at $R = 25$.

The notional amount invested in every CDO tranche is $1.00. Consider the questions:

(a) What are the corresponding default probabilities?
(b) How would you use this information in predicting defaults?
(c) Suppose the defaults are *uncorrelated*. What is the distribution of the number of defaults during one year?
(d) How much would the 0–33% tranche lose under these conditions?
(e) Suppose there are three tranches:

- 0–33%
- 33–66%
- 66–100%

How much would each tranche pay over a year?
(f) Suppose the default correlation goes up to 50%, answer questions (c) – (e) again.

7. Consider the following news from Reuters:

1008 GMT [Dow Jones] LONDON—SG recommends selling 7-year 0–3% tranche protection versus buying 5-year and 10-year 0–3% protection. 7-year equity correlation tightened versus 5-year and 10-year last year. SG's barbell plays a steepening of the 7-year bucket, as well as offering positive roll down, time decay, and jump to default.

SG also thinks Alstom's (1022047.FR) 3–5-year curve is too steep, and recommends buying its 6.25% March 2010 bonds versus 3-year CDS.

(a) What is a barbell? What is positive roll down, time decay?
(b) What is jump to default?
(c) Explain the logic behind SG's strategy.

8. We consider a reference portfolio of four investment grade names with the following one-year CDS rates:

$$c(1) = 56$$
$$c(2) = 80$$
$$c(3) = 137$$
$$c(3) = 12$$

The recovery rate is the same for all names at $R = 25$.

The notional amount invested in every CDO tranche is $100. Consider the questions:

(a) What are the corresponding default probabilities?
(b) How would you use this information in predicting defaults?
(c) Suppose the defaults are uncorrelated, what is distribution of the number of defaults during one year?
(d) How much would the 0–33% tranche lose under these conditions?
(e) Suppose there are three tranches:

- 0–33%
- 33–66%
- 66–100%

How much would each tranche pay over a year?
(f) Let iTaxx(t) be the index of CDS spreads at time t, where each name has a weight of .25. How can you calculate the mezzanine *delta* for a 1% change in the index?
(g) Suppose the default correlation goes up to 50%, answer questions (1)–(4) again.

9. Consider the following news from Reuters:

HVB Suggests Covered Bond Switches

0843 GMT [Dow Jones] LONDON—Sell DG Hyp 4.25% 2008s at 6.5 bp under swaps and buy Landesbank Baden-Wuerttemberg(LBBW) 3.5% 2009s at swaps-4.2 bp, HVB says. The LBBW deal is grandfathered and will continue to enjoy state guarantees; HVB expects spreads to tighten further in the near future.

(a) What is a German Landesbank? What are their ratings?
(b) What is the logic behind this credit strategy?
(c) Can you take the same position using CDSs? Describe how.

10. Explain the following position using appropriate graphs. In particular, make sure that you define a barbell in credit sector. Finally, in what sense is this a convexity position?

1008 GMT [Dow Jones] LONDON—SG recommends selling 7-year 0–3% tranche protection versus 5-year and 10-year 0–3% protection. 7-year equity correlation tightened versus 5-year and 10-year last year.

SG's barbell plays a steepening of the 7-year bucket, as well as offering positive roll down, time decay, and jump to default.